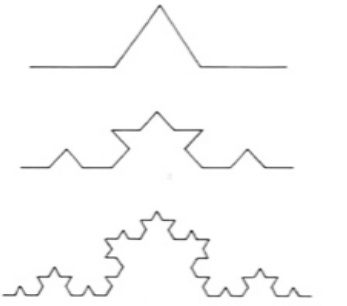
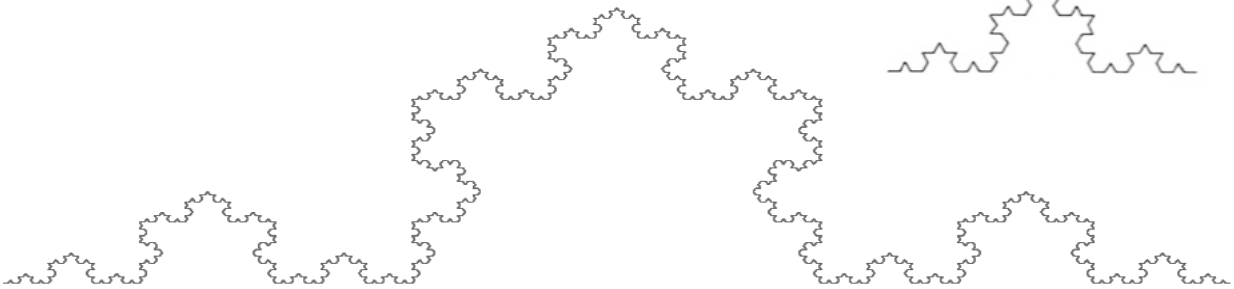
The Koch Curve Fractal

Teacher's Intro

OVERVIEW

Students will learn to transform a simple generator shape into a complex fractal curve. They will repeat the same operation at smaller and smaller scales, and measure the perimeter at each step. The resulting curve is highly complex, has a large perimeter, and is roughly similar to natural fractals such as coastlines.





NM Math Standards:

1.G.3.2 Create simple symmetrical shapes and pictures.
1.G.3.3 Recognize and describe the symmetric characteristics of designs
1.G.4.1 Use combinations of shapes to make a new shape to demonstrate relationships between shapes
2.G.4.4 Relate geometric ideas to numbers (e.g., seeing rows in array as a model of repeated addition).
1.M.2.1 Measure with multiple copies of units the same size
1.M.2.2 Use repetition of a single unit to measure something larger than the unit
2.M.1.1 Identify a unit of measure (e.g., nearest inch) and repeat that unit comparing it to the item being measured.
3.G.4.1 Visualize, build, and draw geometric objects.
4.M.1.1 Select the appropriate type of unit for measuring perimeter and size of an angle.
4.M.1.3 Identify the inverse relationship between the size of the units and the number of units.
5.M.2.3 Apply strategies and use tools for estimating and measuring the perimeter of regular and irregular shapes.

NM Science Standards:

Know that Earth's features are constantly changed by a combination of slow and rapid processes that include the action of volcanoes, earthquakes, mountain building, biological changes, erosion, and weathering. (3'd)

The Koch Curve Fractal

The Koch Curve is a fractal that starts with a simple pattern made of a line that is divided into 3 equal parts.

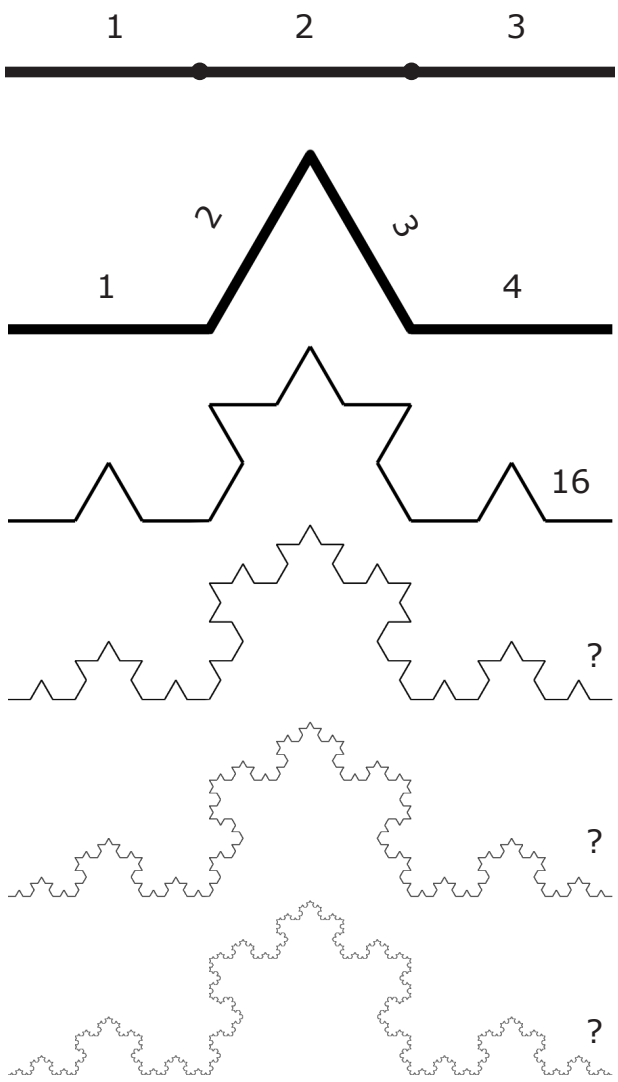
Erase the middle segment and replace it with an upsidedown "V" shape, and now the whole pattern is made up of four line segments.

Next, we do the same thing again. Each of those four lines is divided in thirds, and the middle segment is replaced with a "V". There are now 4x4 or 16 line segments.

Fractals are never-ending patterns made by repeating the same idea over again. So next, we'll replace each of the 16 line segments with the same pattern again.

Each time we do this, the curve gets more jagged and complicated, and its length - or perimeter - gets bigger.

Eventually, the pattern starts to look like a fractal in nature, such as a coastline, or part of a snowflake.



On the next page, you'll get to draw the Koch Curve Fractal using a pencil to replace each segment with a smaller version of the starting pattern. To do this, you'll have to divide each line segment into thirds - 3 equal parts - and then replace the middle segment with the V shape. Repeat again, smaller and smaller. Each time you do this, the curve becomes 1/3 of the original size, and there are 4 times as many of them.

The Koch Curve Fractal

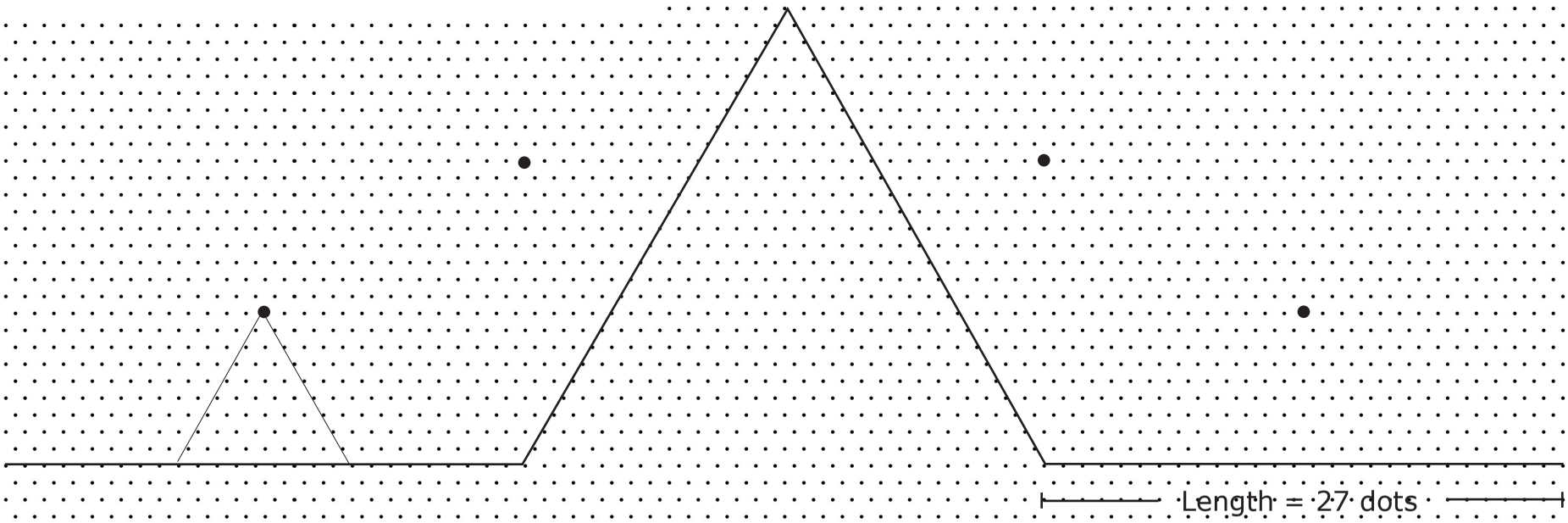
The first version of the shape, Iteration 1, has been drawn for you, and consists of 4 segments, each 27 units long, for a total length of 108

Using a pencil, divide each segment into 3 equal parts, then replace the middle third of the segment with an upsidedown "V". This is Iteration 2.

Count how many segments you have now, and figure out how long each of these segments is. Multiply the number of segments by the length of each to find the total length of the Koch Curve.

Repeat this process to make Iterations 3 and 4, each time erasing the middle segment and replacing it with a smaller copy of the "V". Fill in the values in the table for Iteration 3 and 4.

Iteration	Number of segments	Length of each segment	Total Length
1	4	27	108
2			
3			
4			



Fractals are SMART: Science, Math & Art!

www.FractalFoundation.org

The Koch Curve Fractal

The Koch Curve fractal is interesting because it is similar to some fractals found in nature. In some ways, the Koch Curve looks like the coastline of a continent or an island. Coastlines are usually not smooth, straight lines, but instead they have lots of similar detail at different sizes. Coastlines are fractals.

How long is the coastline of Britain (shown to the right)? The answer is: it depends how closely you measure it! With a very small ruler, the length, or perimeter, is VERY large, as more and more details appear at small scales. Imagine measuring every rock, pebble and grain of sand on the beach!

Why are coastlines fractals? Because they are formed by simple, repetitive processes, over thousands or millions of years. The crashing of waves slowly erodes the coastline. So does the rising and falling of the tides. Giant storms also erode the coastline, leaving behind fractal patterns. These shapes are much more irregular than the Koch Curve, but they are both formed in similar ways.

Another naturally occurring fractal that the Koch Curve reminds us of is hinted at below. If you take 3 copies of the Koch Curve, rotate them and combine them as shown, you end up with a 6-fold symmetric object... that looks like a snowflake! Snowflakes grow by expanding outward from the center, and branching again and again. It's not exactly the same process as the Koch Curve, but they are both made by doing a simple thing over and over again. Like all fractals, complex shapes come from simple repetition.